Scott Brickman

University of Maine at Fort Kent, USA

brickman@maine.edu

Collection Vectors and the Octatonic Scale

ABSTRACT

Background

Allen Forte, writing in *The Structure of Atonal Music* (1973), tabulated the prime forms and vectors of pitch-class sets (PCsets). Additionally, he presented the idea of a set-complex, a set of sets associated by virtue of the inclusion relation. Post-Tonal Theorists such as John Rahn, Robert Morris and Joseph Straus, allude to Forte's set-complex, but do not treat the concept at length.

The interval vector, developed by Donald Martino ("The Source Set and Its Aggregate Formations," *Journal of Music Theory* 5, no. 2, 1961) is a means by which the intervals, and more specifically the dyads contained in a collection, can be identified and tabulated.

There exists no such tool for the identification and tabulation of collections other than dyads that are contained in larger collections.

My presentation will be a study of the properties of a set, commonly known as the octatonic scale. The study will examine the properties and relationships among the subsets of the octatonic scale. The goals of the presentation will be twofold: first, to construct a model that summarizes the properties of the subsets of the octatonic scale in a "collection vector", and second, to theorize about pitch-class set networks that allow the navigation from one octatonic hexachord to another.

As a 12-tone composer who works almost exclusively with ordered rows whose initial hexachords are subsets of the octatonic scale, I found that much of my pre-compositional work was redundant in that it replicated that which I had used in previous pieces. I wanted to collate all the information I had accrued regarding the octatonic collection.

I therefore codified the subsets, supersets, properties and relationships among octatonic collections.

The octatonic scale is sometimes commonly know as the "half-whole" scale or the diminished scale. It is called the "half-whole" scale because of its alternation of half and whole steps between adjacent pitches of the scale. The two forms of the ocatonic collection are 1) c-c#-d#-e-f#-g-a-bb-C and 2) c-d-eb-f-f#-g#-a-b-c. It is referred to as the "diminished scale" because the non-adjacent pitches form fully diminished seventh chords.

Using set theory designations, developed by Allen Forte, the octatonic scale (collection) is 8-28 (0134679T). There is one seven-pitch subset of 8-28, which is 7-31 (0134679). There are six hexachords that are subsets of the octatonic scale: 6z13 (013467), 6z23 (023568), 6-27 (013469), 6-30 (013679), 6z49 (013479) and 6z50 (014679).

6z13 transforms to 6z50 under the M7 operation. 6z23 and 6z49 map unto themselves under the M7 operation, as do 6-27 and 6-30 whose compliments are themselves.

So, a way to perceive these hexachords is that they provide five areas.

The interval vector, developed by Donald Martino, tabulates the intervals (dyads) in a collection. In order to comprehend the constitution of the octatonic hexachords, I decided to create a "collection vector" which would tabulated the trichordal, tetrachordal and pentachordal subsets of each octatonic hexachord.

I began exploring the properties of the OH by tabulating their trichordal subsets. What my "collection" or "subset" vector showed, was that among the seven trichords that are subsets of the octatonic scale, 3-5 (016 or the "Maria" trichord) and 3-8 (026) were the most common subsets.

There are thirteen tetrachords that are subsets of the octatonic scale. Not surprisingly, the most common are 4z15 and 4z29, the two and only two "z" related tetrachords. Other than those, 4-12 (0236), 4-18 (0147) and 4-27 (0258) are heavily weighted tetrachords. Unlike the tricordal subsets, which are distributed rather evenly amongst the six OH, the tetrachordal distribution is unevenly weighted. 4-28 (0369) the fully diminished seventh chord)is a subset of only two hexachords, which is somewhat surprising given the "diminished" nature of the octatonic collection.

Like the trichord subsets, the seven pentachords are distributed among the octatonic hexachords rather evenly. 5-19 (01367) and 5-28 (02368) are the most common. So, just as an interval vector tabulates dyads in a set, a simple extention of this concept is the collection vector, which tabulates subsets of a collection.

Most importantly, I was able to make some generalizations regarding octatonic collections. Every octatonic hexachord contains all seven of the octatonic trichords (3-2 (013), 3-3 (014), 3-5 (016), 3-7 (025), 3-8 (026), 3-10 (036), and 3-11 (037). As stated previously, 3-5 (016 or the "Maria" trichord) and 3-8 (026) are the most common.

6z13, 6z23, 6-27 and 6-30 share 4-12 (0236) and 4-13 (0136) as second most common tetrachords; additionally, 6z49 has 4-12, 6z50 has 4-13. Every octatonic hexachord has 4-18 except 6z23. Every OH has 4-27 (0258) except 6z13.

Most significant and interesting, is that 6-30 is the hexachord that contains the wealth of the most common octatonic subsets. It is in effect, a referential hexachord for octatonic identity.

Aims and repertoire studied

Categories of Sets

Methods

The inclusion property

Implications

Set networks

Keywords

Set Theory

REFERENCES

Berger, Arthur. *Reflections of an American Composer*, Berkeley: University of California Press, 2002.
Forte, Allen. *The Structure of Atonal Music*, New Haven: Yale University Press, 1973.
Lewin, David. *Musical Form and Transformation*, Oxford: Oxford University Press, 2007.
Martino, Donald. "The Source Set and its Aggregate Formations." JMT 5/2 (1961): 1-16.
Morris, Robert. *Composition with Pitch Classes*, New Haven: Yale University Press, 1987.
Rahn, John. *Basic Atonal Theory*, New York: Schirmer Books, 1980.
Rimsky-Korsakov, Nikolay. *My Musical Life*, 3rd ed., New York: Alfred A. Knopf, 1942.
Straus, Joseph N., *Introduction to Post-Tonal Theory*, 4th ed., New York: W. W. Norton, 2016.
).